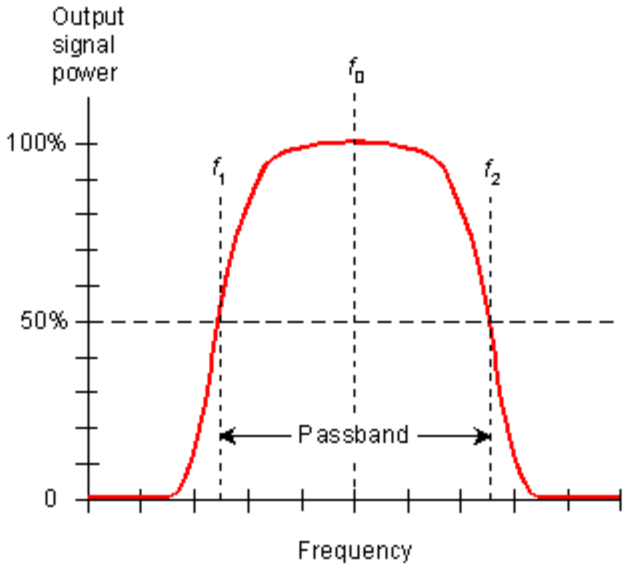
Group Delay

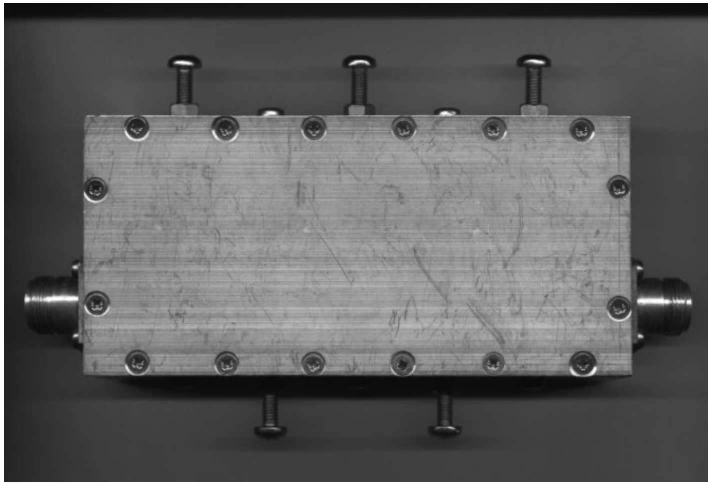
Ron Hranac

The Basics

Bandpass filter equivalent

- **Consider the 6 MHz spectrum occupied by an analog TV channel or digitally modulated signal, the 5-42 MHz upstream spectrum, or any specified bandwidth or passband as the equivalent of a bandpass filter.**





The Basics

Transit time and velocity of propagation

- **A signal takes a certain amount of time to pass through a filter**

  The transit time through the filter is a function of the filter's velocity of propagation (also called velocity factor)

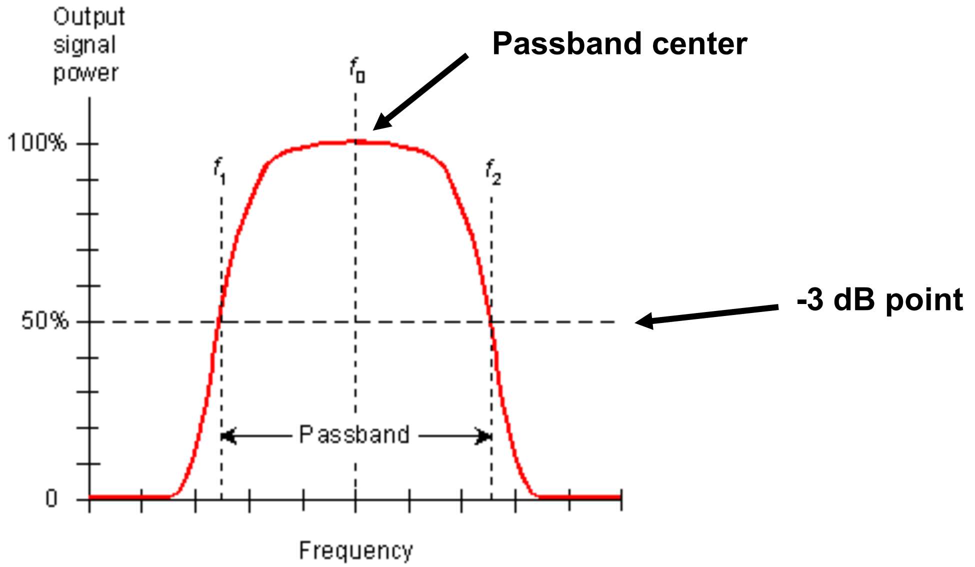
  Velocity of propagation is the speed that an electromagnetic signal travels through some medium relative to the speed of light in a vacuum, expressed as a percentage (e.g., 87%)

  Velocity factor is the speed that an electromagnetic signal travels through some medium relative to the speed of light in a vacuum, expressed in decimal format (e.g., 0.87)

The Basics

Velocity of propagation versus frequency

- **In many instances the velocity of propagation through a filter varies with frequency**
  - **The velocity of propagation may be greater in the center of the filter's passband, but slower near the band edges**



The Basics

Delay and absolute delay

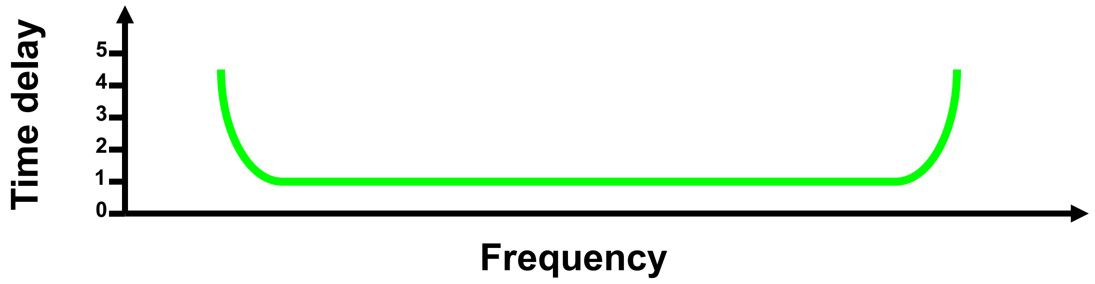
- The finite time required for a signal to pass through a filter—or any device or component for that matter—is called delay

Absolute delay at a given frequency is the delay a continuous-wave (CW) signal experiences passing through the device. As an example, consider that the delay through a device, with a given CW frequency, corresponds to two and one-eighth cycles of the sinusoid, which is 765 degrees (two cycles are 360° × 2 = 720° plus one-eighth cycle of 45° equals 765°). Note that when measuring the phase shift of the sinusoid and comparing the input signal to the device with the output signal of the device, the measurement will be 45°, since the signal is a CW sinusoid, even though the actual phase shift due to the delay is 765°. In this example, 765° is the actual cumulative phase shift introduced into the CW signal due to the time delay through the device.

The Basics

Delay versus frequency

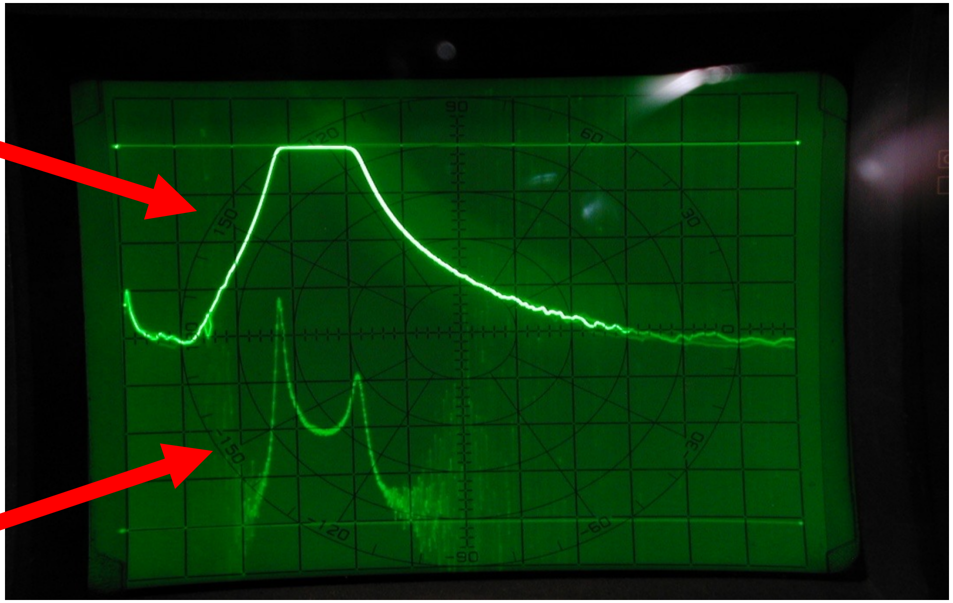
- If delay through a filter is plotted on a graph of frequency (x-axis) versus time delay (y-axis), the plot often has a parabola- or bathtub-like shape



The Basics

Network analyzer plot of Ch. T8 bandpass filter

- The upper trace shows magnitude versus frequency: the filter's bandpass characteristics. The x-axis is frequency, the y-axis is amplitude.
- The lower trace shows group delay versus frequency. The x-axis is frequency, the y-axis is time. Note the bathtub-like shape of the curve.



Graphic courtesy of Holtzman, Inc.

The Basics

Phase versus frequency

- If propagation or transit time through a device is the same at all frequencies, phase is said to change proportionally with respect to frequency. A graph of the cumulative phase shift-versus-frequency for the device is a straight line (with slope proportional to the transit time) when the transit time is the same at all frequencies.
- If phase changes proportionally with frequency, an output signal will be identical to the input signal – except that it will have a time shift because of the uniform delay through the device.
- If propagation or transit time through a device is different at different frequencies, the result is a nonlinear phase shift-versus-frequency characteristic for the device.
- If phase does not change proportionally with frequency, the output signal will be distorted due to the “nonlinear phase distortion.” Note that even though the terminology is “nonlinear phase distortion,” the device in this case is still operating as a “linear device” and is NOT undergoing what is often called “nonlinear distortion.”

The Basics

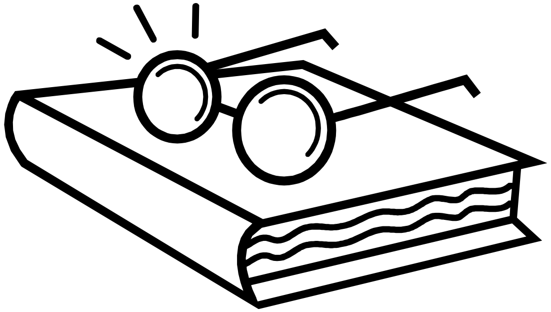
Delay and phase distortion

- Delay distortion—also known as phase distortion—is usually expressed in units of time: millisecond (ms), microsecond (µs) or nanosecond (ns) relative to a reference frequency
- Phase distortion is related to phase delay
- Phase distortion is measured using a parameter called envelope delay distortion, or group delay distortion

The Formal Definition

Group delay is "the [negative] derivative of radian phase with respect to radian frequency. It is equal to the phase delay for an ideal non-dispersive delay device, but may differ greatly in actual devices where there is a ripple in the phase versus frequency characteristic."

IEEE Standard Dictionary of Electrical and Electronics Terms



The Math (Yikes!)

- **In its simplest mathematical representation…**

$$GD = -\frac{d\varphi}{d\omega}$$

where *GD* is group delay variation, *φ* is phase in radians, and *ω* is frequency in radians per second

- **Group delay τ also is defined**

$$\tau(\omega) = -\frac{\partial\varphi(\omega)}{\partial\omega}$$

- **And yet another definition is**

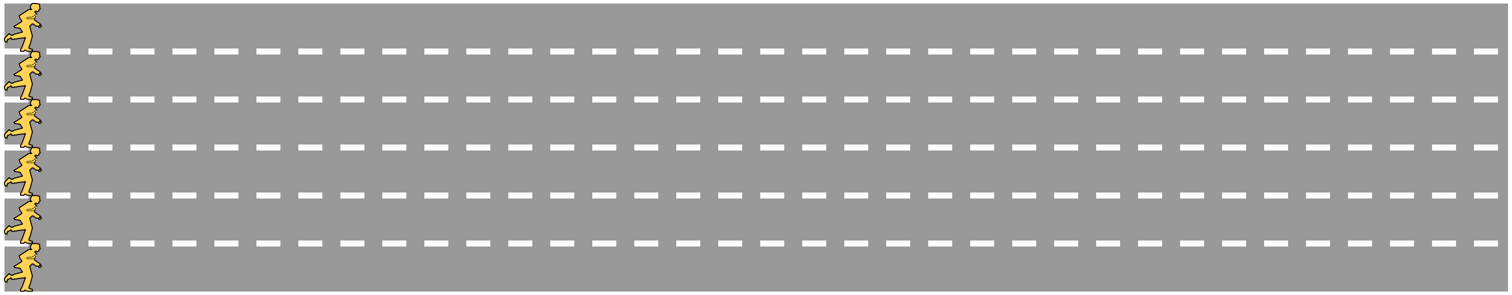
$$D(\omega) \underline{\underline{\Delta}} -\frac{d}{d\omega}\Theta(\omega) \underline{\underline{\Delta}} -\frac{d}{d\omega}\angle H(e^{j\omega \mathrm{T}})$$

The Translation (Whew!)

- If phase versus frequency is non-linear, group delay exists.
- In a system, network, device or component with no group delay, all frequencies are transmitted through the system, network, device or component in the same amount of time—that is, with equal time delay.
- If group delay exists, signals at some frequencies travel faster than signals at other frequencies.

The Analogy

Imagine a group of runners with identical athletic abilities on a smooth, flat track …

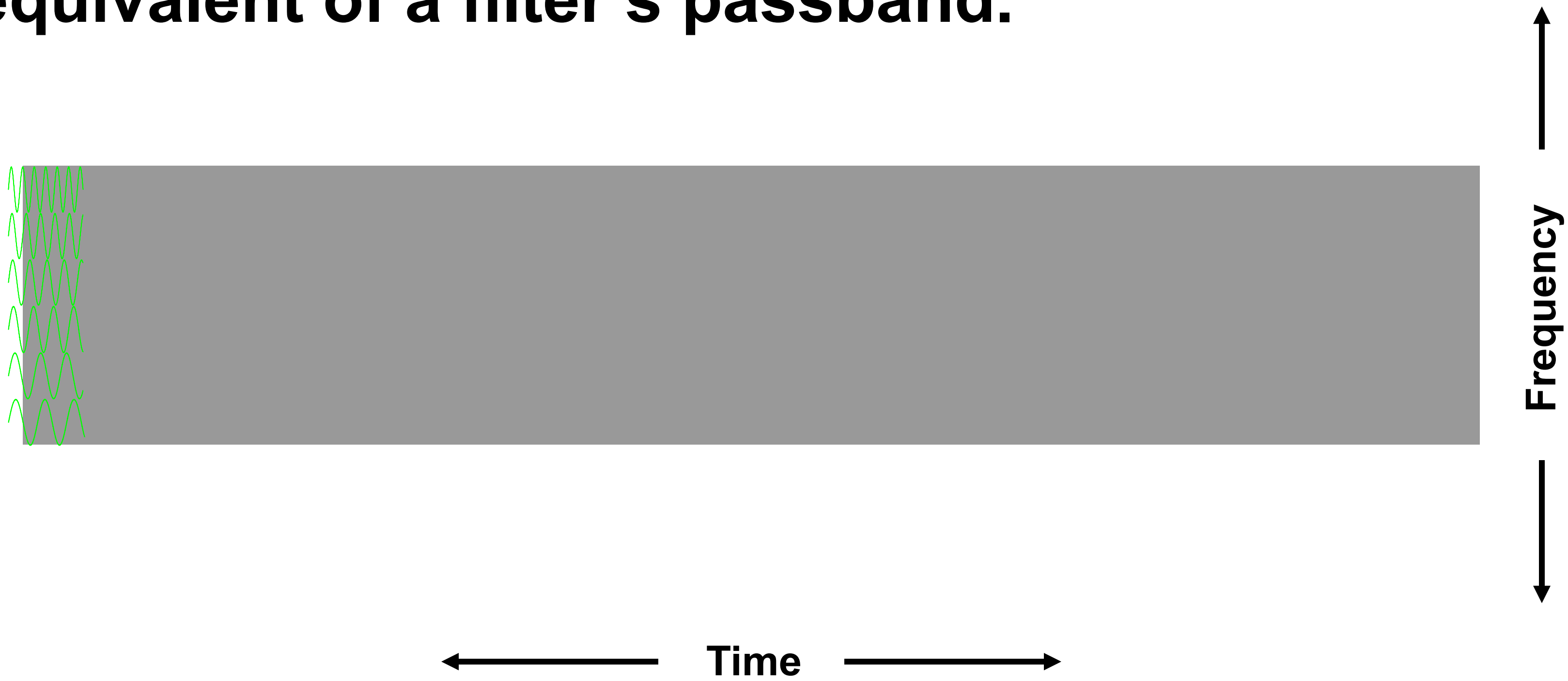


All of the athletes arrive at the finish line at exactly the same time and with equal time delay from one end of the track to the other!

Example courtesy of Holtzman, Inc.

Group Delay: An Analogy

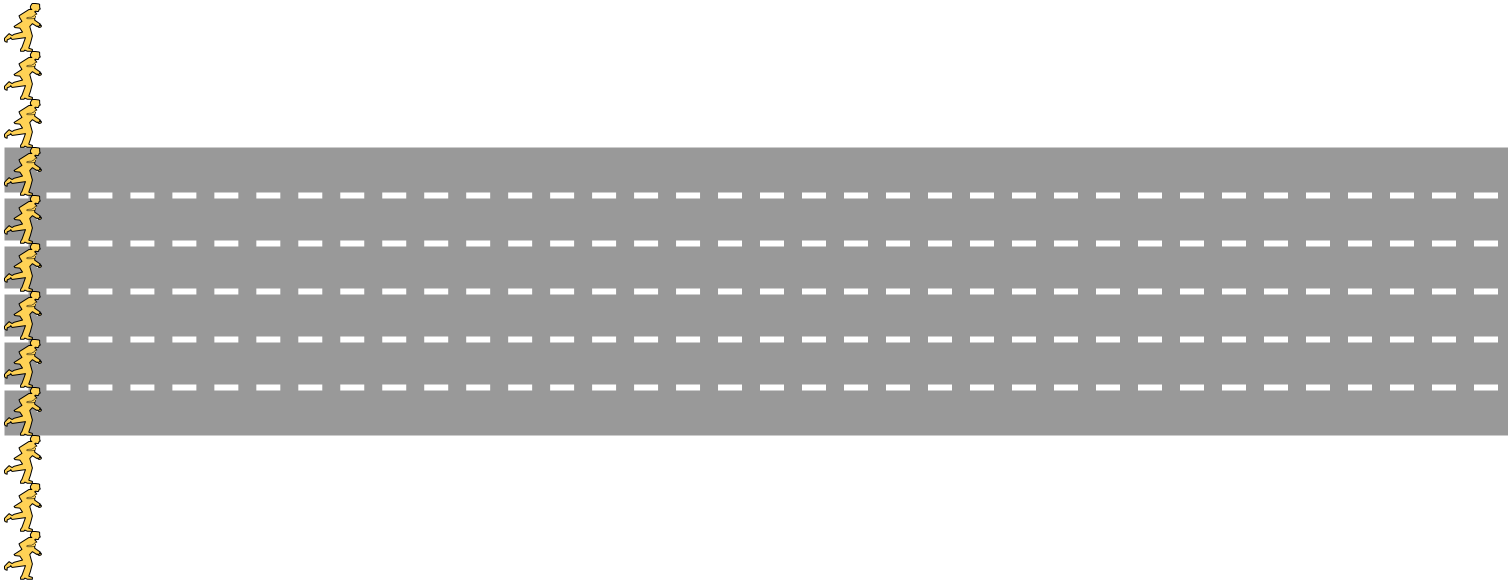
Now let's substitute a group of RF signals for the athletes. Here, the "track" is the equivalent of a filter's passband.



All of the frequencies arrive at the destination at exactly the same time and with equal time delay through the filter passband!

Group Delay: An Analogy

Back to athletes, but now there are some that have to run in the ditches next to the track.

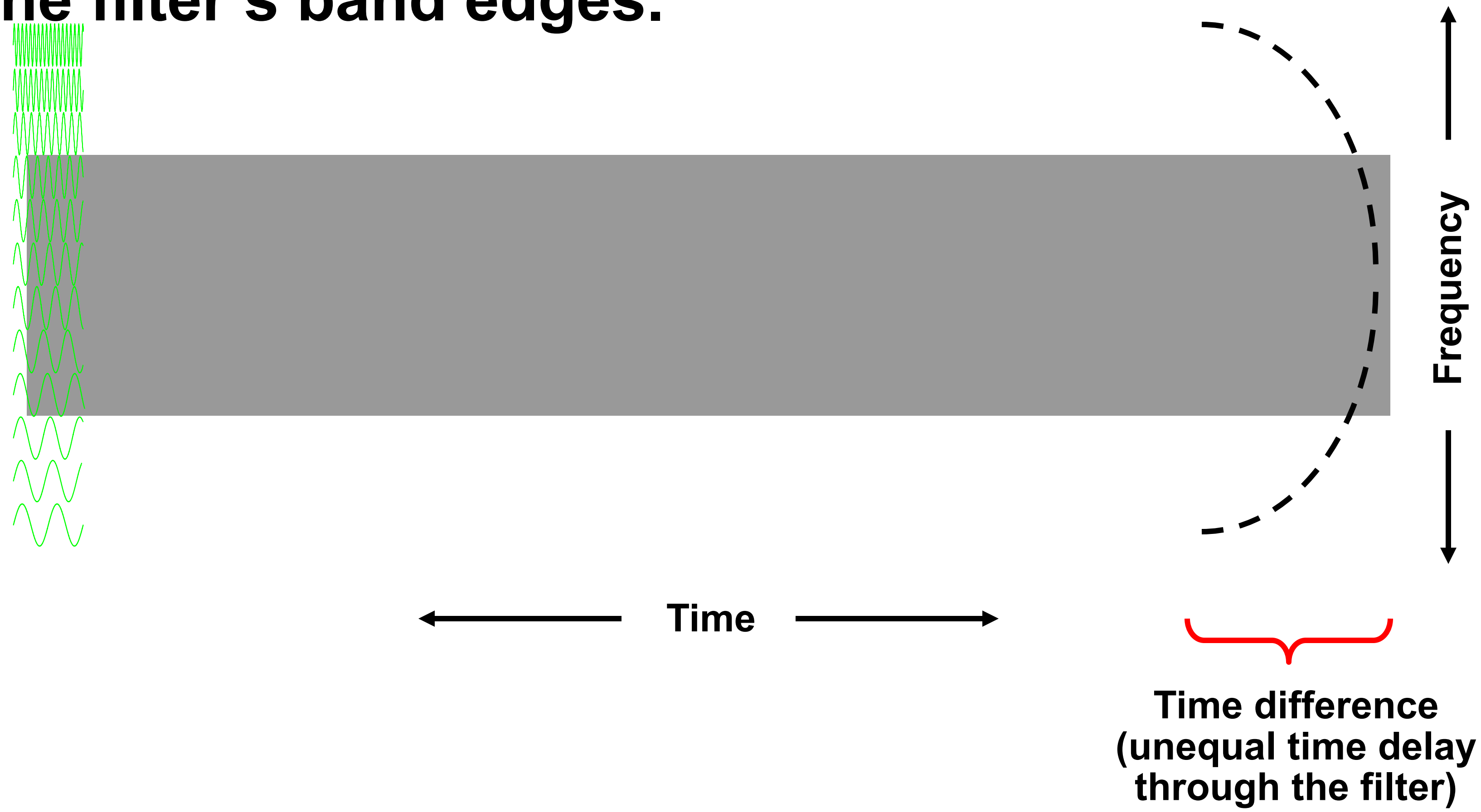


Some athletes take a little longer than others to arrive at the finish line. Their time delay from one end of the track to the other is unequal.

Example courtesy of Holtzman, Inc.

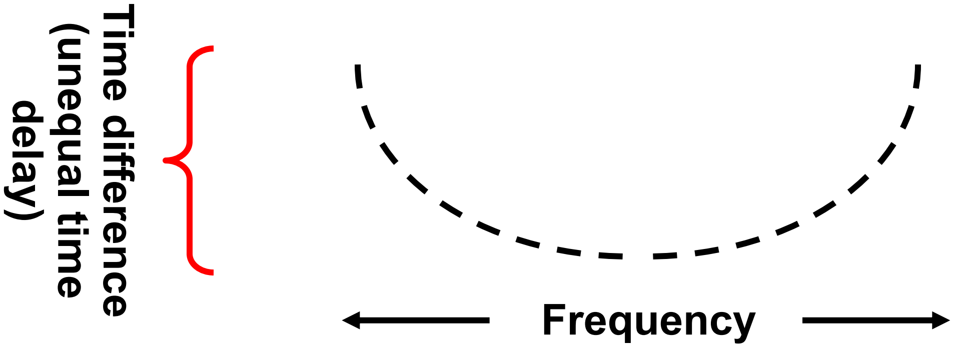
Group Delay: An Analogy

Substitute RF signals for the athletes again. The "track" is a filter's passband, the "ditches" are the filter's band edges.



Group Delay: An Analogy

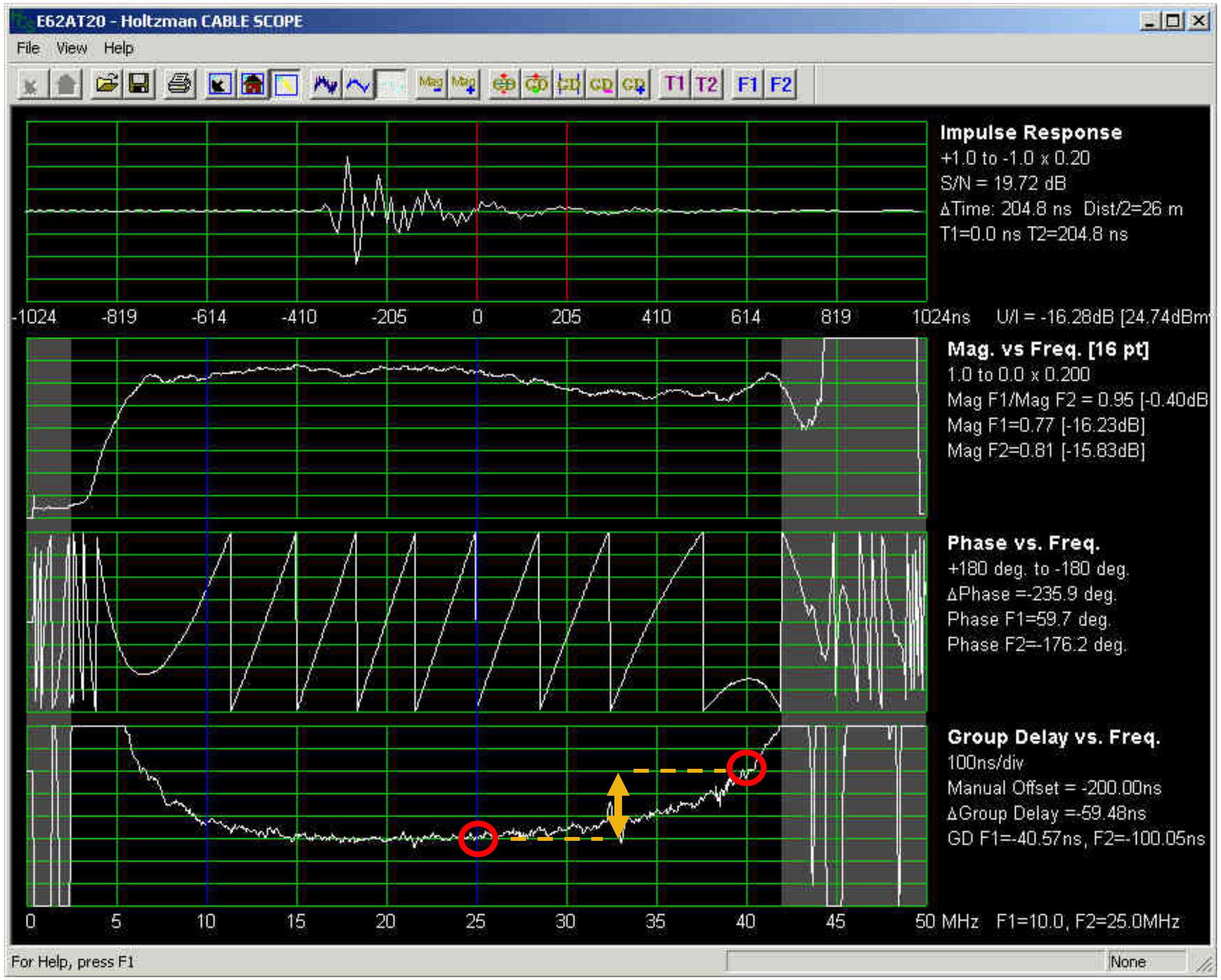
- **Group delay exists, because some frequencies—the ones near the band edges—took longer than others to travel through the filter!**
- **Now take the dotted line connecting the frequencies and flip it on its side. The result is the classic bathtub-shaped group delay curve.**



The Classic Group Delay "Bathtub Curve"

- In this example, the group delay between 25 and 40 MHz is about 300 ns (3 vertical divisions at 100 ns each)
- That is, it takes the 40 MHz signal 300 ns longer to reach the headend than the 25 MHz signal

Time difference in nanoseconds

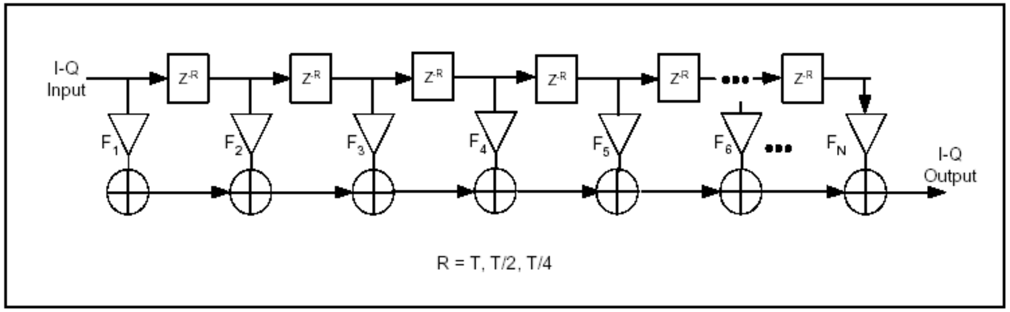


Frequency

Graphic courtesy of Holtzman, Inc.

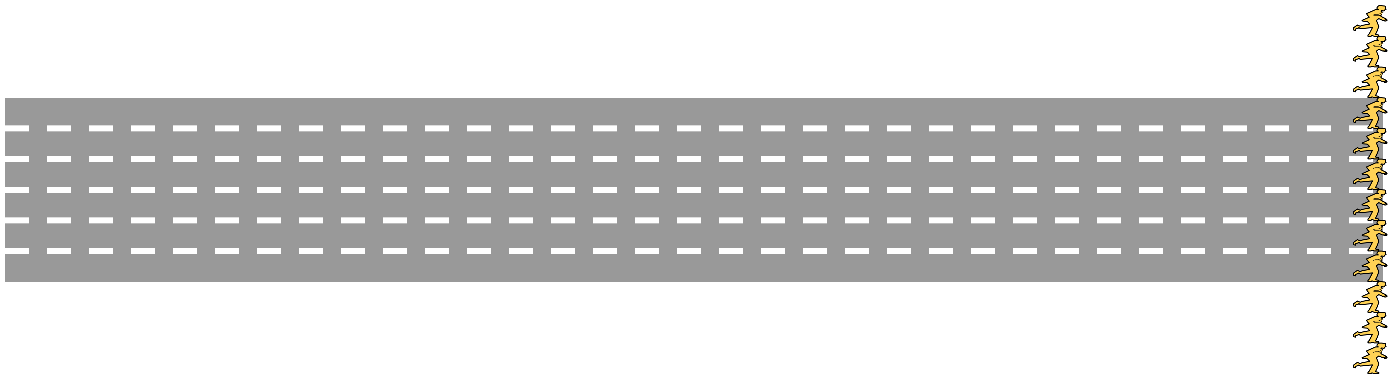
Adaptive Equalization

- **Adaptive equalization is a tool to combat linear distortions such as group delay**
- **All DOCSIS cable modems support downstream adaptive equalization (always on)**
- **DOCSIS 1.1 and later cable modems support upstream adaptive equalization (pre-equalization of the transmitted signal, may be turned on/off by cable operator)**
  - **DOCSIS 1.1 modems: 8-tap adaptive pre-equalization**
  - **DOCSIS 2.0 and later modems: 24-tap adaptive pre-equalization**
  - **Adaptive pre-equalization is not generally supported in D1.0 modems**



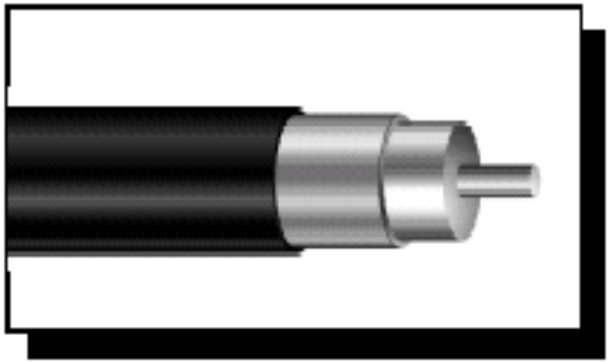
Adaptive Equalization

- **Recall the group delay analogy using athletes on a track:**
  - **Adaptive equalization can be thought of as analogous to delaying the runners in the middle of the track, allowing the slower runners in the "ditches" to catch up.**
  - **This allows all runners to arrive at the finish line at the same time, with equal time delay.**



Phase Versus Frequency

- OK, group delay exists if phase versus frequency is non-linear
- But just what does that mean?
- Let's look at an example, using a 100 ft piece of .500 feeder cable



Phase Versus Frequency

Back to Basics: Velocity of Propagation

- Hardline coax used for feeder applications has a velocity of propagation of around 87%

  The speed of light in free space or a vacuum is 299,792,458 meters per second, or 983,571,056.43 feet per second—1 foot in about 1.02 ns

  In coaxial cable with a velocity of propagation of 87%, electromagnetic signals travel at a velocity equal to 87% of the free space value of the speed of light. That works out to 260,819,438.46 meters per second, or 855,706,819.09 feet per second—1 foot in about 1.17 ns

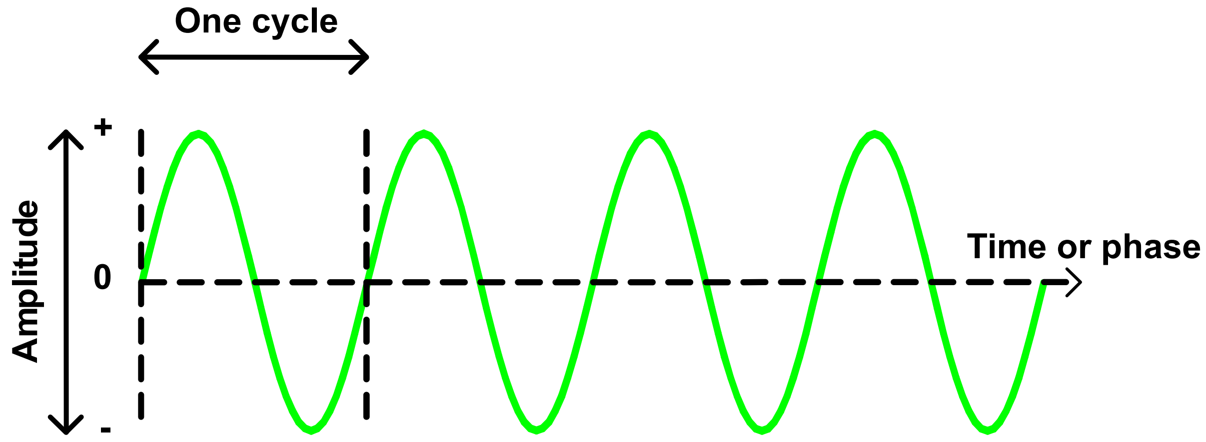
  So, electromagnetic signals will travel 100 ft in a vacuum in 101.67 ns, and through a 100 ft piece of coax in 116.86 ns

Phase Versus Frequency

Back to Basics: Wavelength and Period

- **Wavelength (λ) is the speed of propagation of an electromagnetic signal divided by its frequency (f) in hertz (Hz). It is further defined as the distance a wave travels through some medium in one period.**

  Period (T) of a cycle (in seconds) = 1/f, where f is frequency in Hz



Phase Versus Frequency

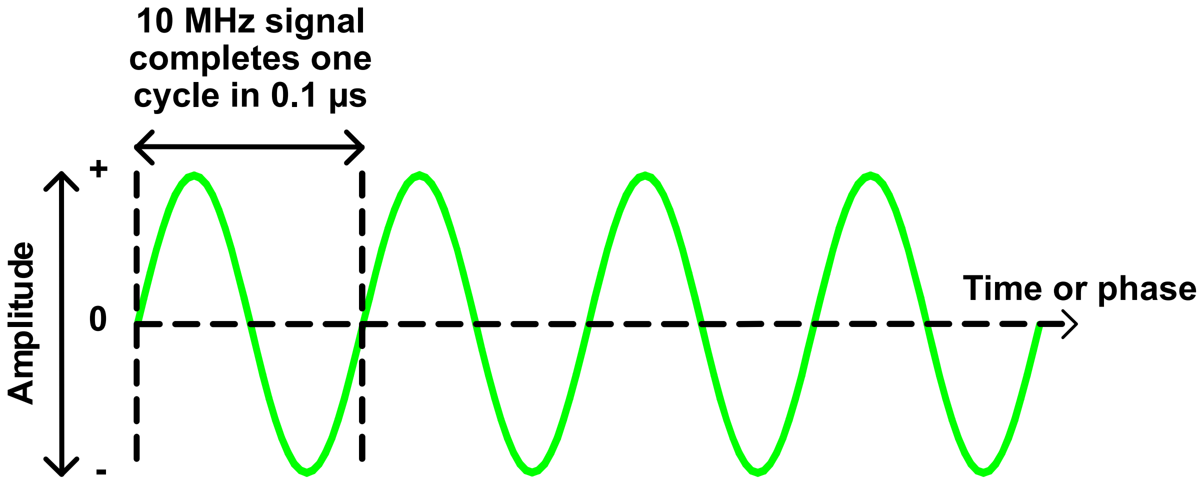
Back to Basics: Wavelength Formulas

- In a vacuum, wavelength in feet (λ_{ft}) = 983,571,056.43/f_{Hz}, which is the same as λ_{ft} = 983.57/f_{MHz}
- In coaxial cable with 87% VP, λ_{ft} = 855,706,819.09 /f_{Hz} or λ_{ft} = 855.71/f_{MHz}

Phase Versus Frequency

Back to Basics: An Example

- For instance, the period of a 10 MHz sine wave is 1/10,000,000 Hz = 1×10^{-7} second, or 0.1 microsecond. That means a 10 MHz signal takes 0.1 µs to complete one cycle, or 1 second to complete 10,000,000 cycles.
- In a vacuum, the 10 MHz signal travels 98.36 ft in 0.1 µs. This distance is one wavelength in a vacuum.
- In 87% velocity of propagation coax, the 10 MHz signal travels 85.57 ft in 0.1 µs. This distance is one wavelength in coax.



Phase Versus Frequency

- **If we can calculate a given frequency's wavelength in feet, we can say that a 100 ft piece of .500 coax is equivalent to a certain number wavelengths at that frequency!**
- **From the previous example, it stands to reason that a 100 ft piece of coax is equivalent to just over one wavelength at 10 MHz. That is, the 10 MHz signal's 85.57 ft wavelength in coax is just shy of the 100 ft overall length of the piece of coax.**

Phase Versus Frequency

- **OK, let's figure out the wavelength in feet for several frequencies in a vacuum and in our 100 ft piece of coax, using the previous formulas. Because of the cable's velocity of propagation, each frequency's wavelength in the cable will be a little less than it is in a vacuum.**

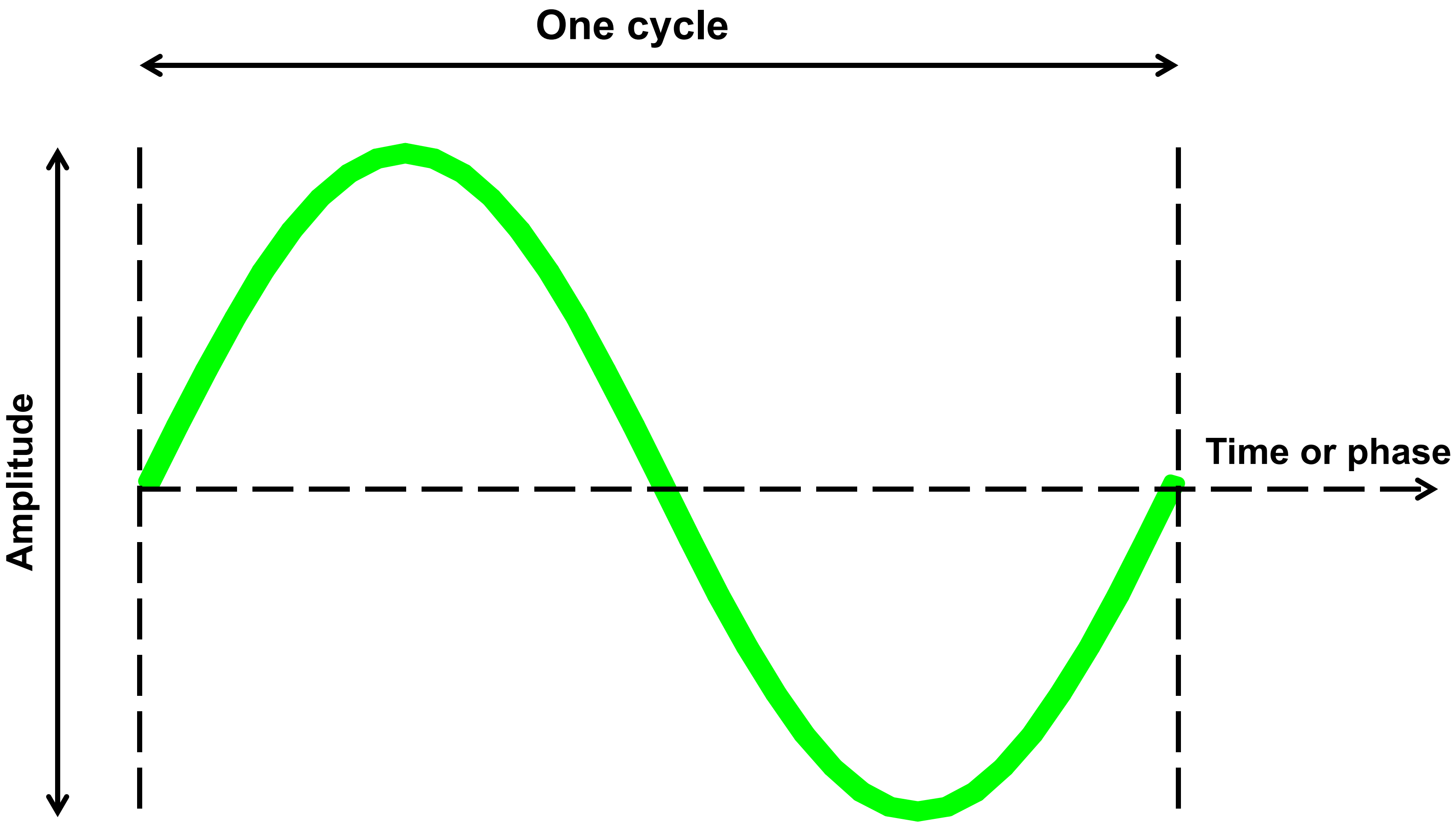
Frequency	λ_{ft} in a vacuum	λ_{ft} in coax
1 MHz	983.57 feet	855.71 feet
5 MHz	196.71 feet	171.14 feet
10 MHz	98.36 feet	85.57 feet
30 MHz	32.97 feet	28.52 feet
42 MHz	23.42 feet	20.37 feet
50 MHz	19.67 feet	17.11 feet
65 MHz	15.13 feet	13.16 feet
100 MHz	9.84 feet	8.56 feet

Phase Versus Frequency

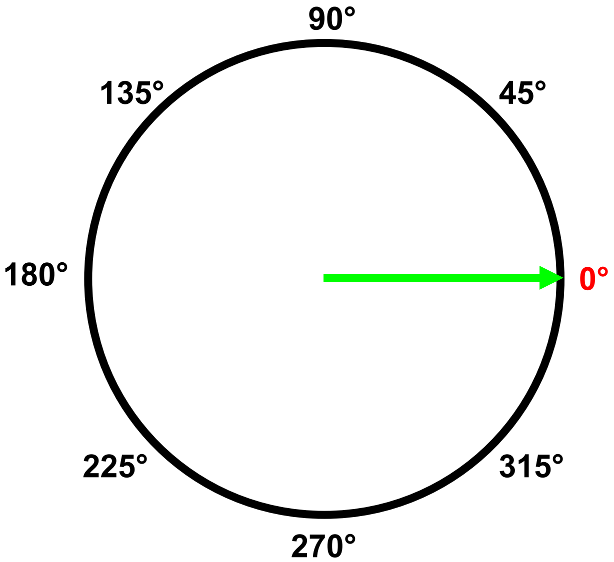
- **Next let's figure out the number of wavelengths for each frequency in the 100 ft piece of coax**

Frequency	λ_{ft} in a vacuum	λ_{ft} in coax	Number of λ in 100 ft of coax
1 MHz	983.57 feet	855.71 feet	0.12 λ
5 MHz	196.71 feet	171.14 feet	0.58 λ
10 MHz	98.36 feet	85.57 feet	1.17 λ
30 MHz	32.97 feet	28.52 feet	3.51 λ
42 MHz	23.42 feet	20.37 feet	4.91 λ
50 MHz	19.67 feet	17.11 feet	5.84 λ
65 MHz	15.13 feet	13.16 feet	7.60 λ
100 MHz	9.84 feet	8.56 feet	11.69 λ

10 MHz Sine Wave: A Closer Look



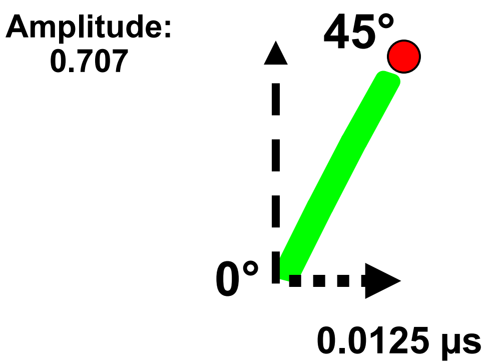
10 MHz Sine Wave: A Closer Look

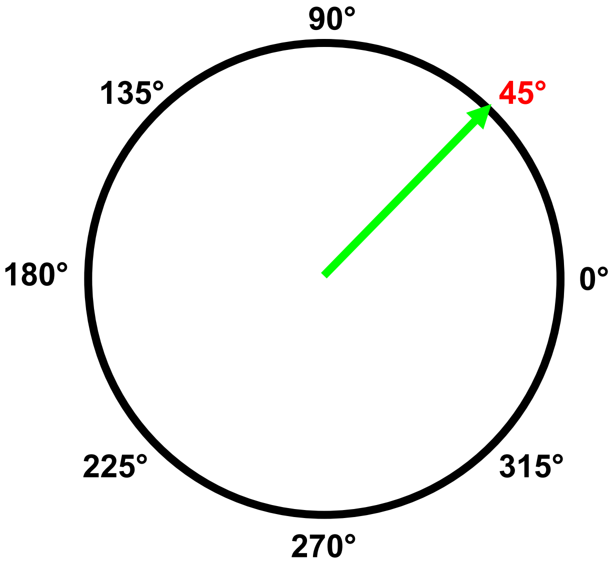


Amplitude:
0

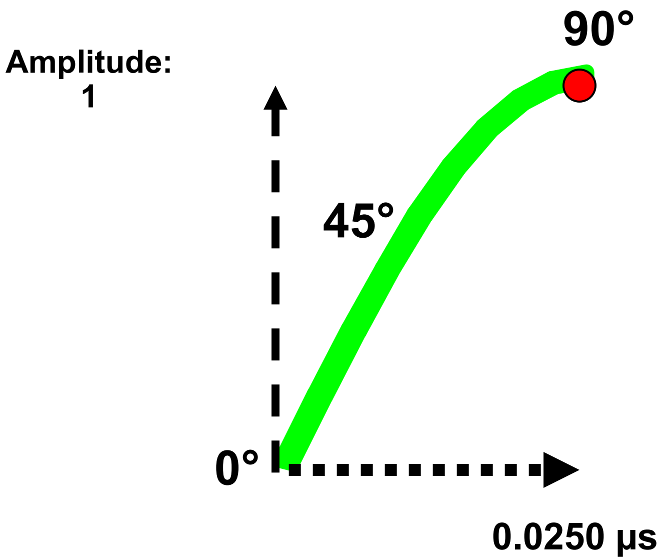
0°

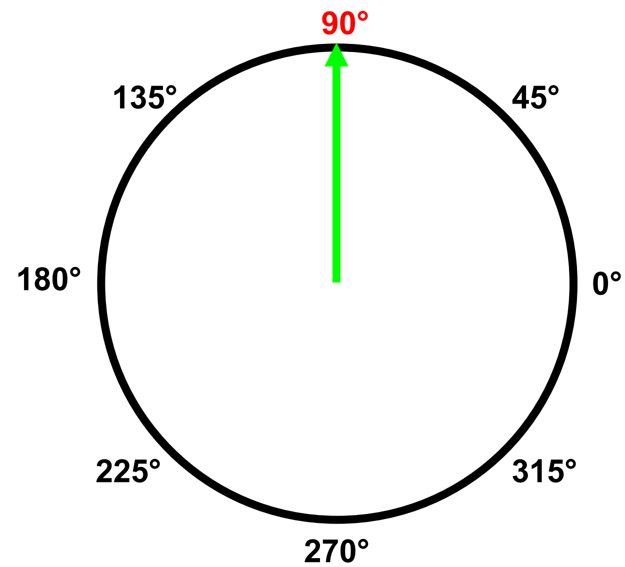
10 MHz Sine Wave: A Closer Look



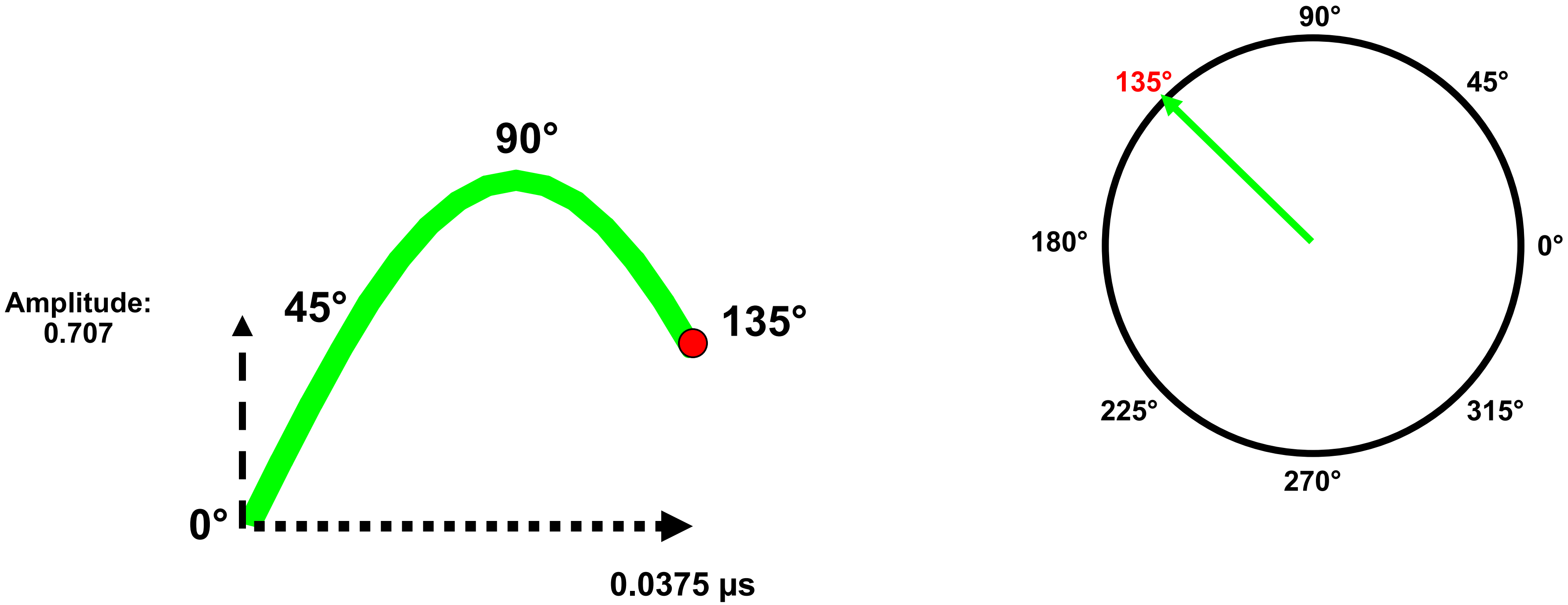


10 MHz Sine Wave: A Closer Look

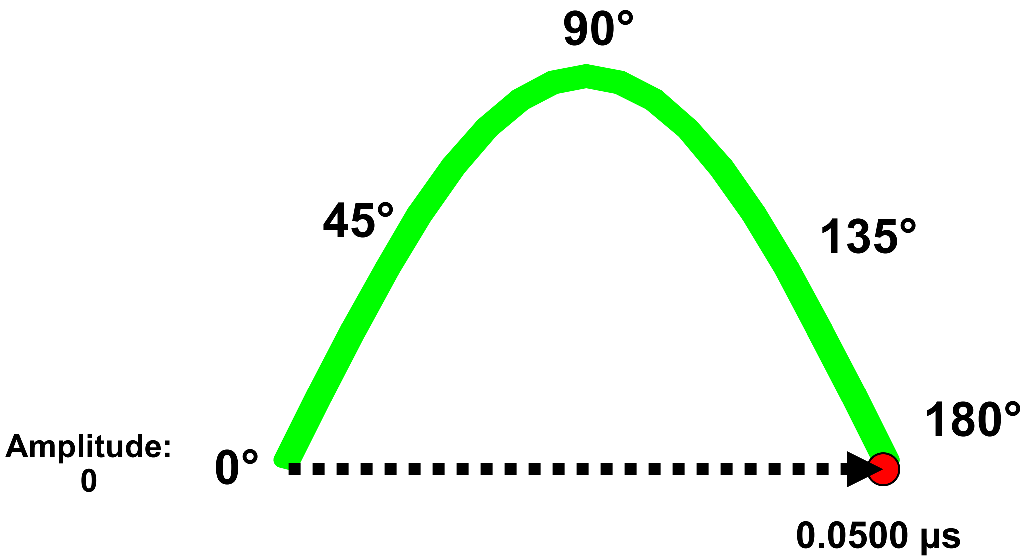


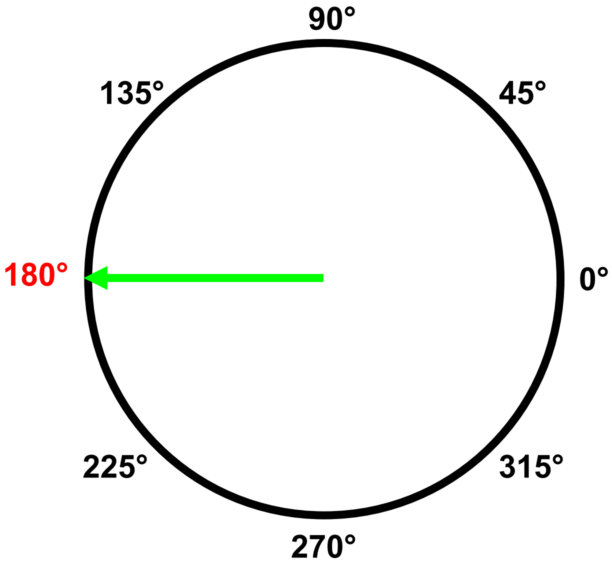


10 MHz Sine Wave: A Closer Look

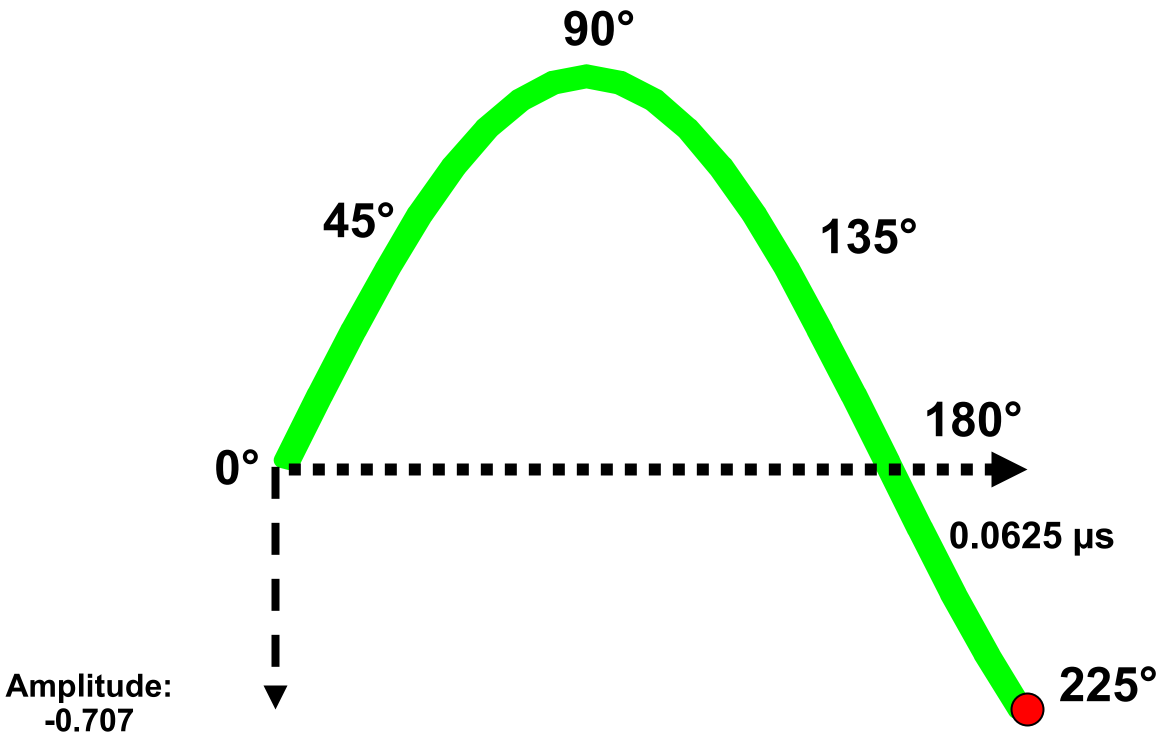


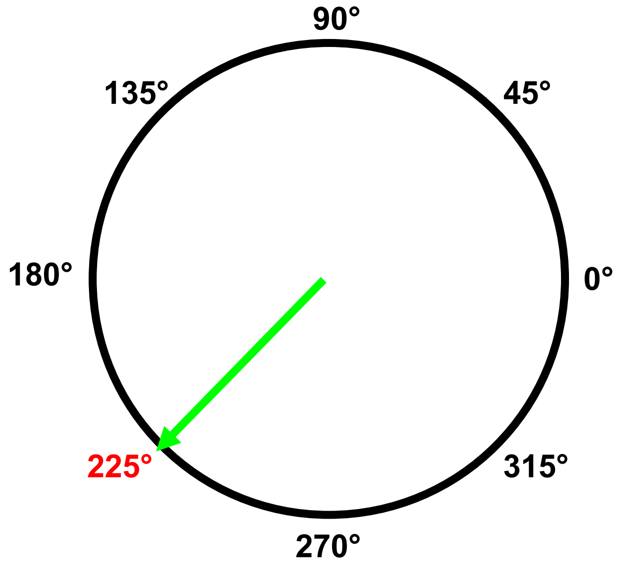
10 MHz Sine Wave: A Closer Look



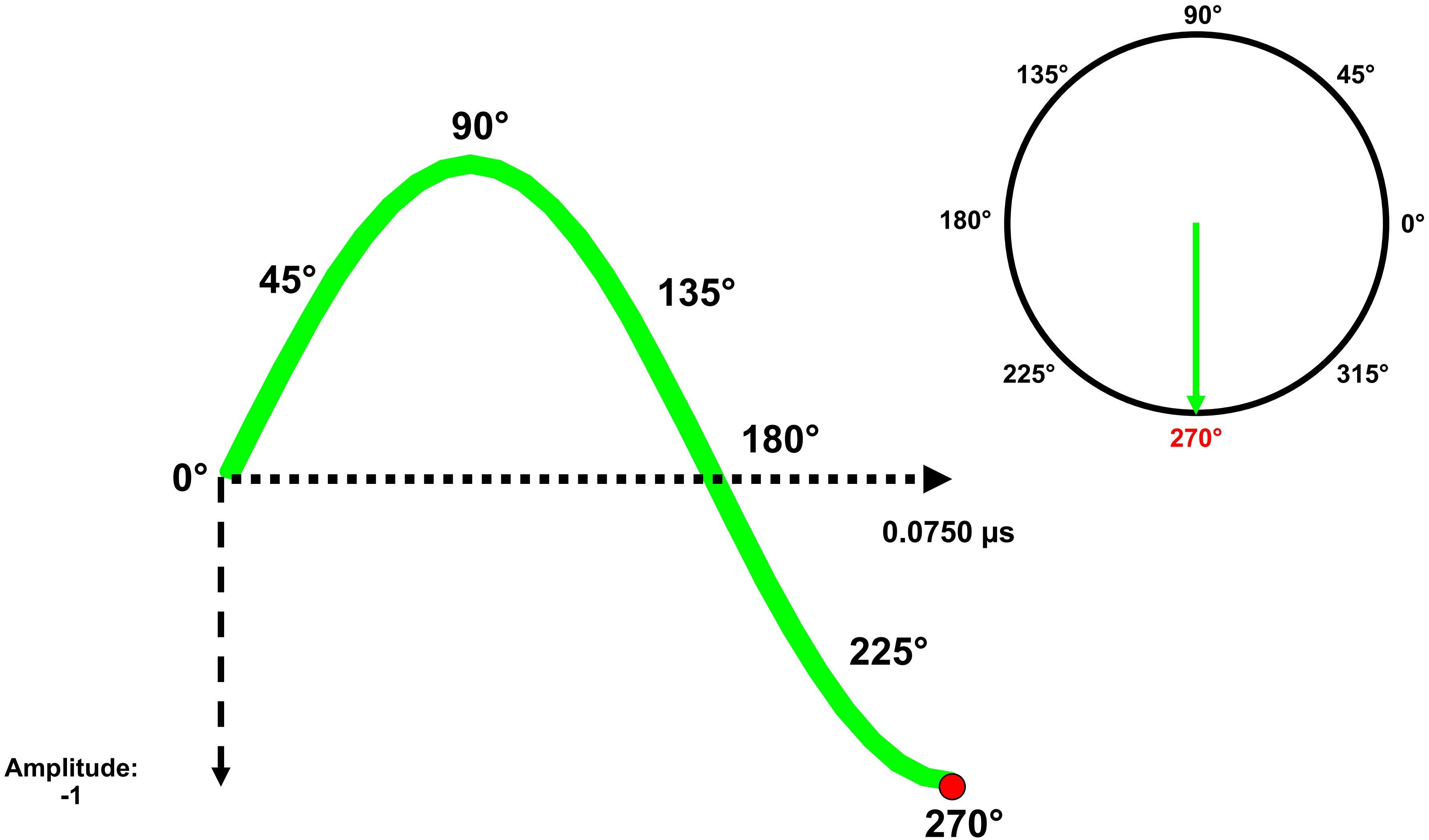


10 MHz Sine Wave: A Closer Look

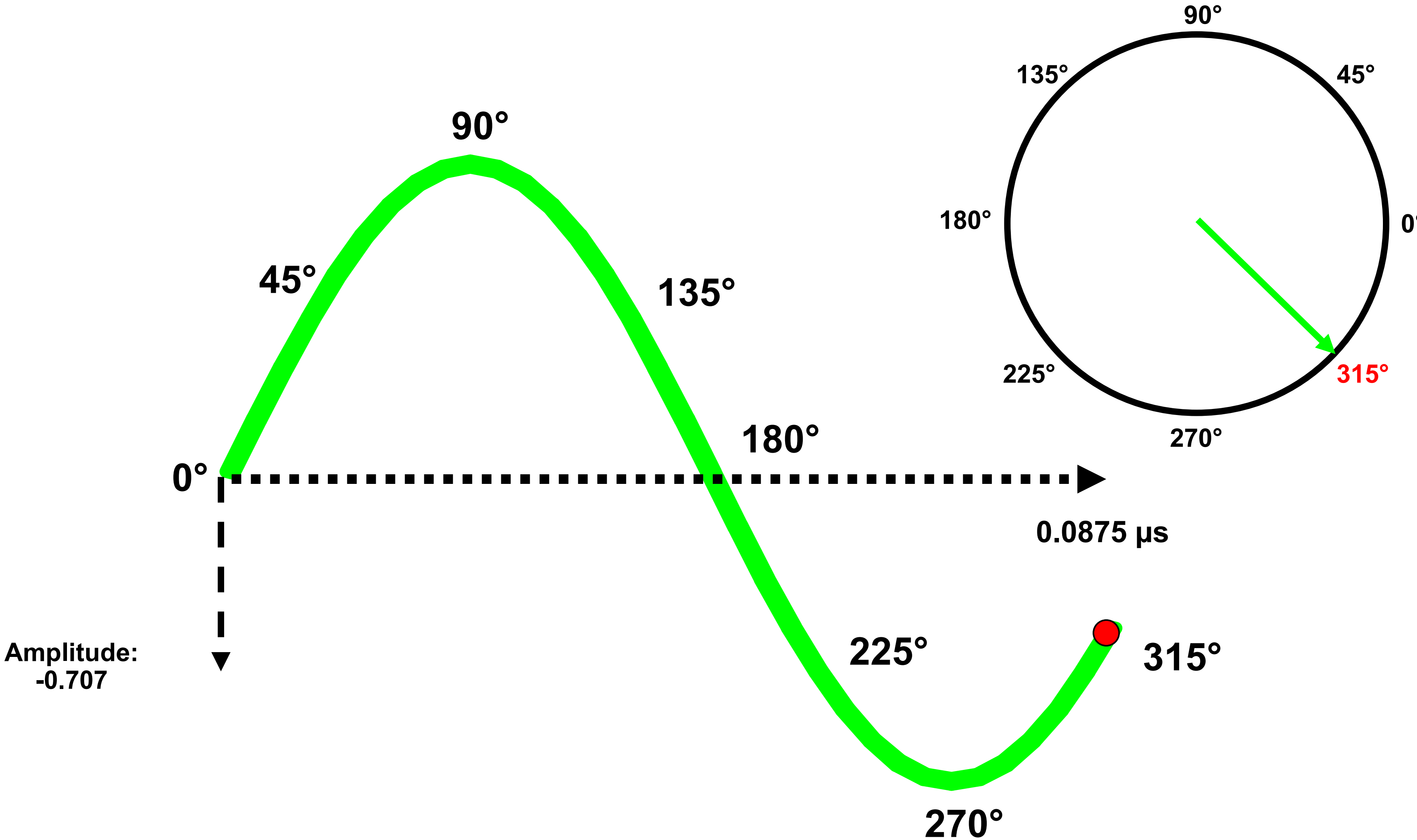




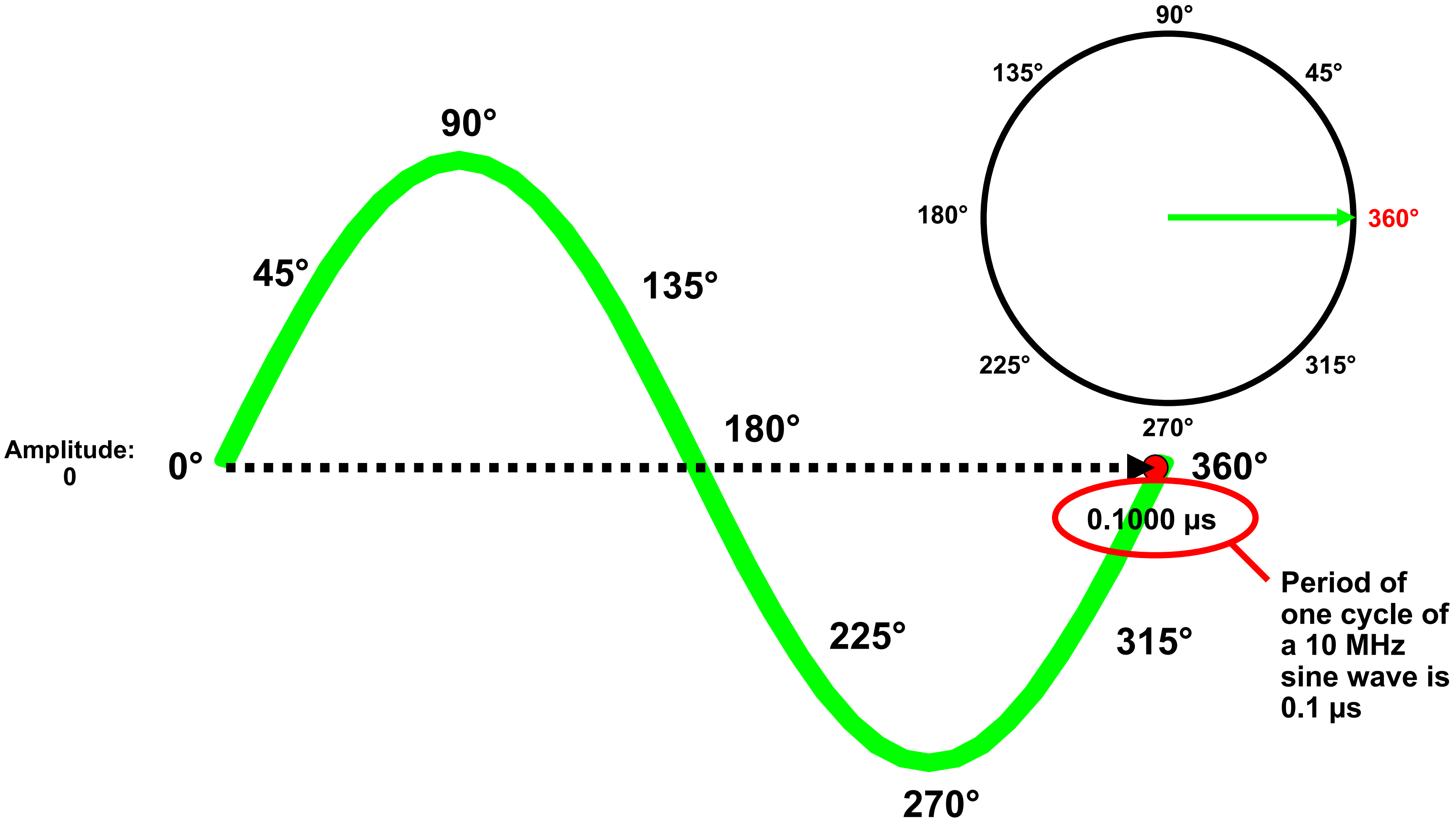
10 MHz Sine Wave: A Closer Look



10 MHz Sine Wave: A Closer Look

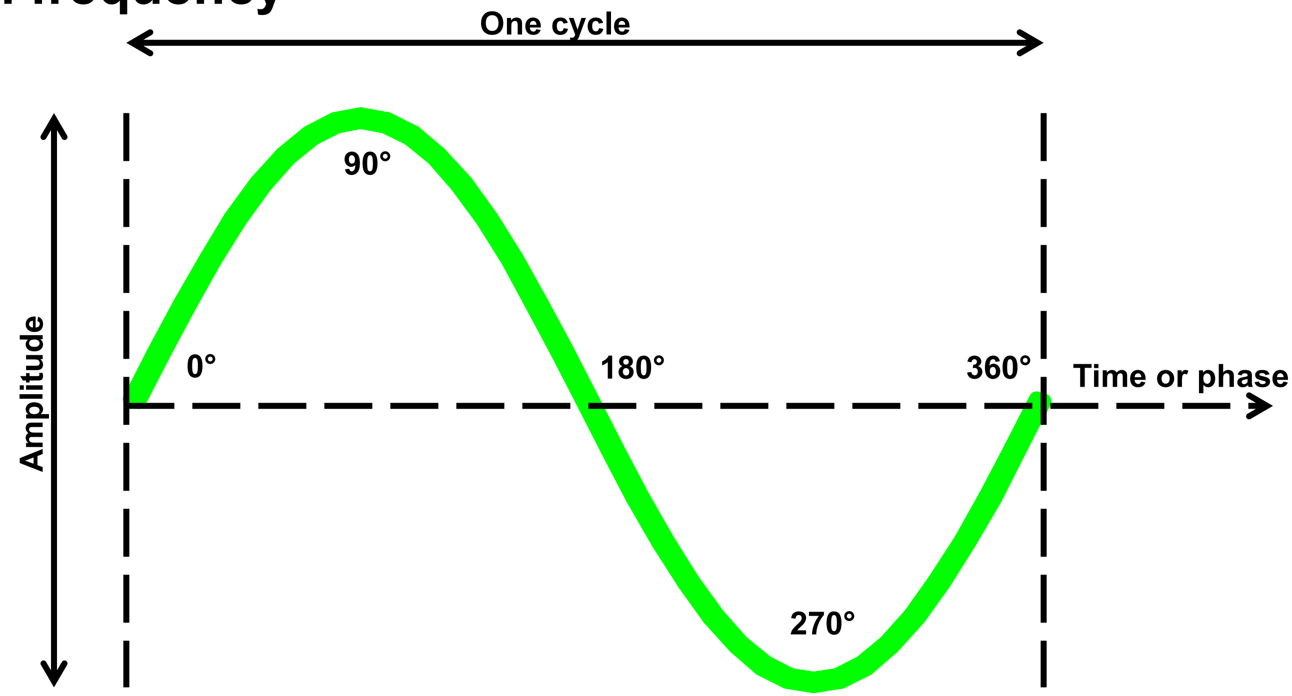


10 MHz Sine Wave: A Closer Look



Phase Versus Frequency

- Knowing that one wavelength (cycle) of a sine wave equals 360 degrees of phase, we can now figure out the total number of degrees of phase the 100 ft piece of cable represents at each frequency

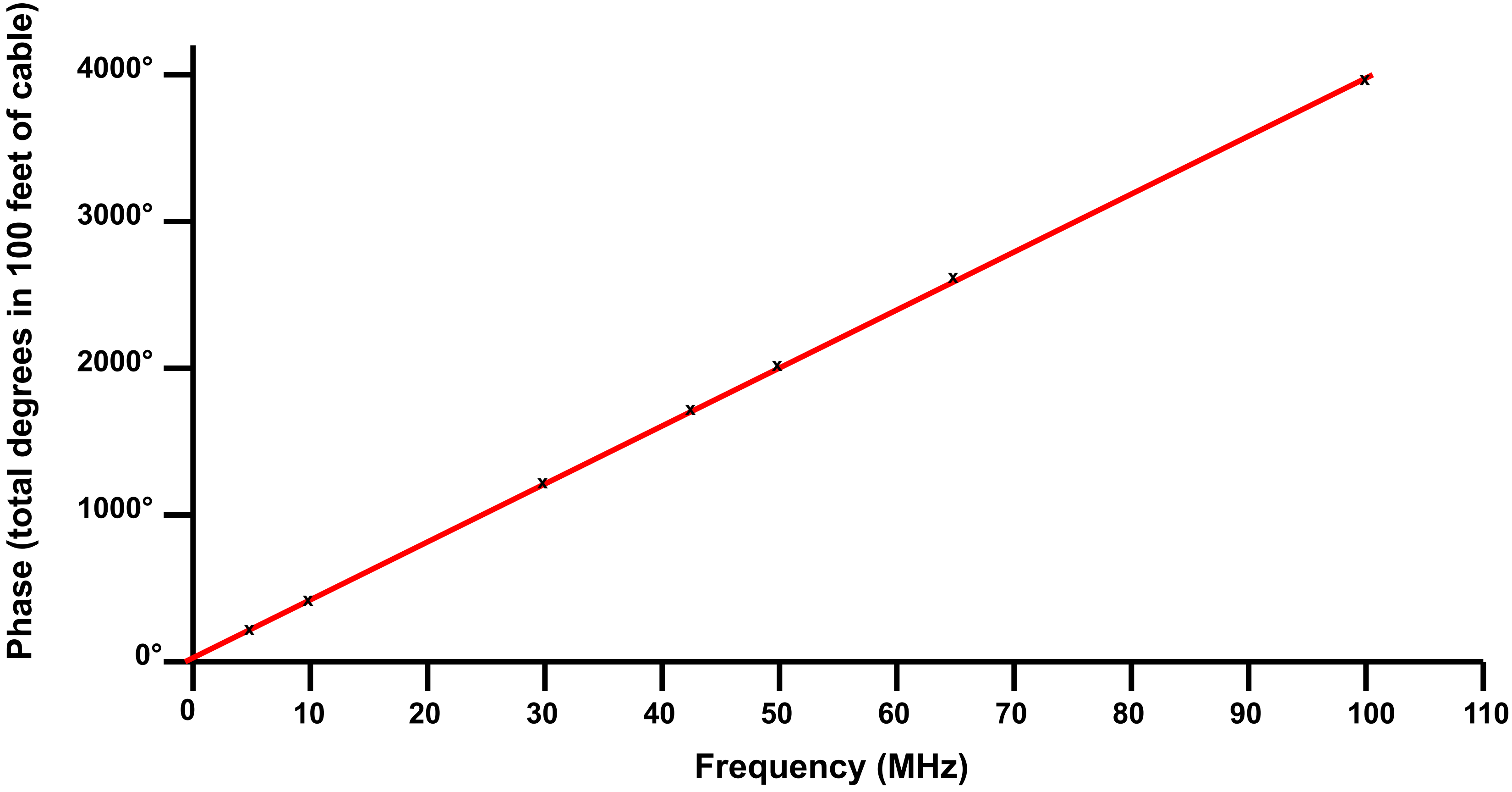


Phase Versus Frequency

Frequency	λ_{ft} in a vacuum	λ_{ft} in coax	Number of λ in 100 ft of coax	Total phase in degrees in 100 ft of coax
1 MHz	983.57 feet	855.71 feet	0.12 λ	42.07°
5 MHz	196.71 feet	171.14 feet	0.58 λ	210.35°
10 MHz	98.36 feet	85.57 feet	1.17 λ	420.71°
30 MHz	32.97 feet	28.52 feet	3.51 λ	1262.12°
42 MHz	23.42 feet	20.37 feet	4.91 λ	1766.96°
50 MHz	19.67 feet	17.11 feet	5.84 λ	2103.53°
65 MHz	15.13 feet	13.16 feet	7.60 λ	2734.58°
100 MHz	9.84 feet	8.56 feet	11.69 λ	4207.05°

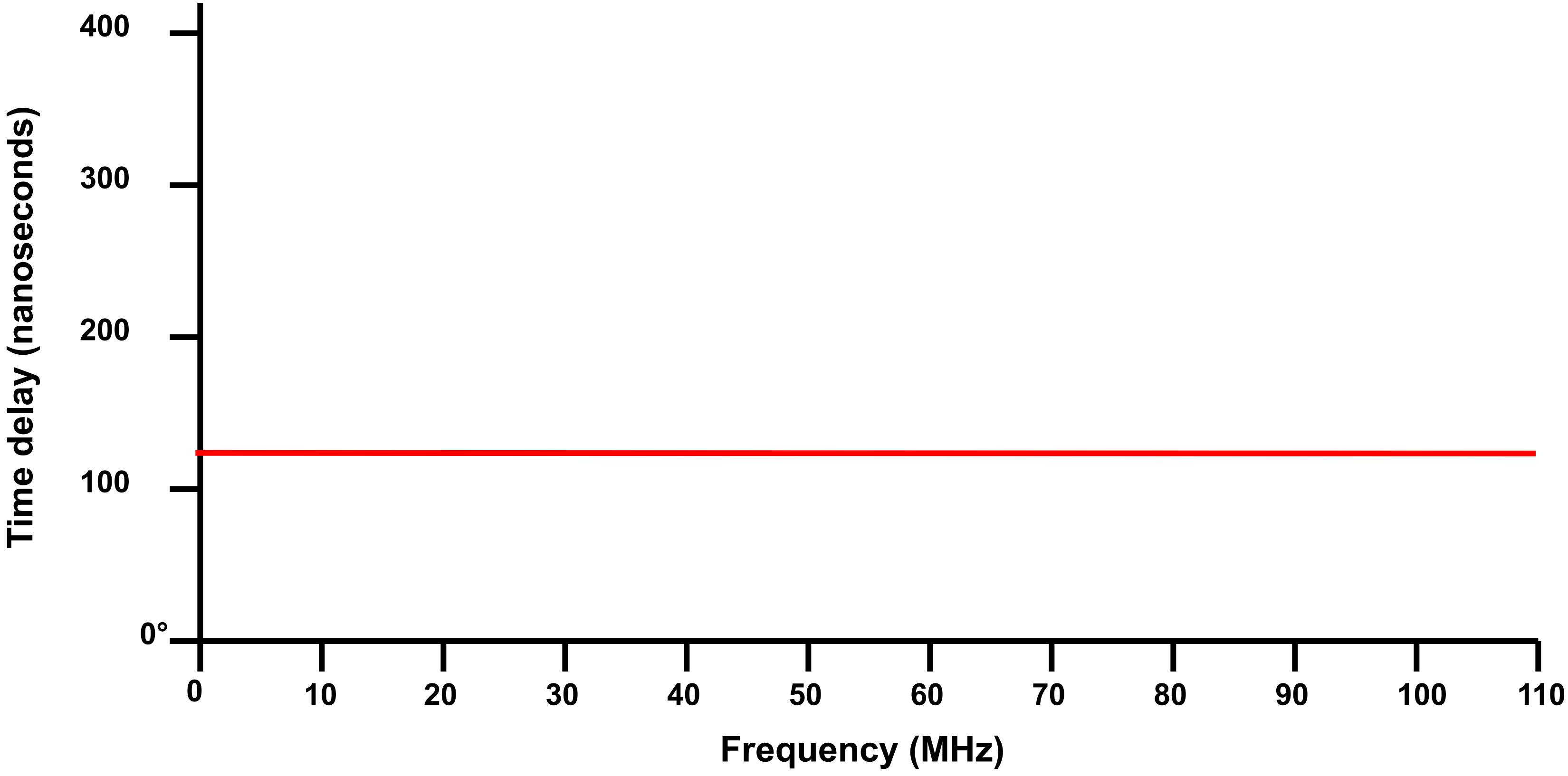
Phase Versus Frequency

- **Next, we can plot the 100 ft piece of cable's phase versus frequency on a graph. In this example, the line is straight—that is, phase versus frequency is linear.**



Phase Versus Frequency

- **Finally, we can plot the time delay for each frequency through the 100 ft piece of cable. This line is the negative of the derivative of radian phase with respect to radian frequency. It’s flat, because phase versus frequency is linear—there is no group delay variation!**

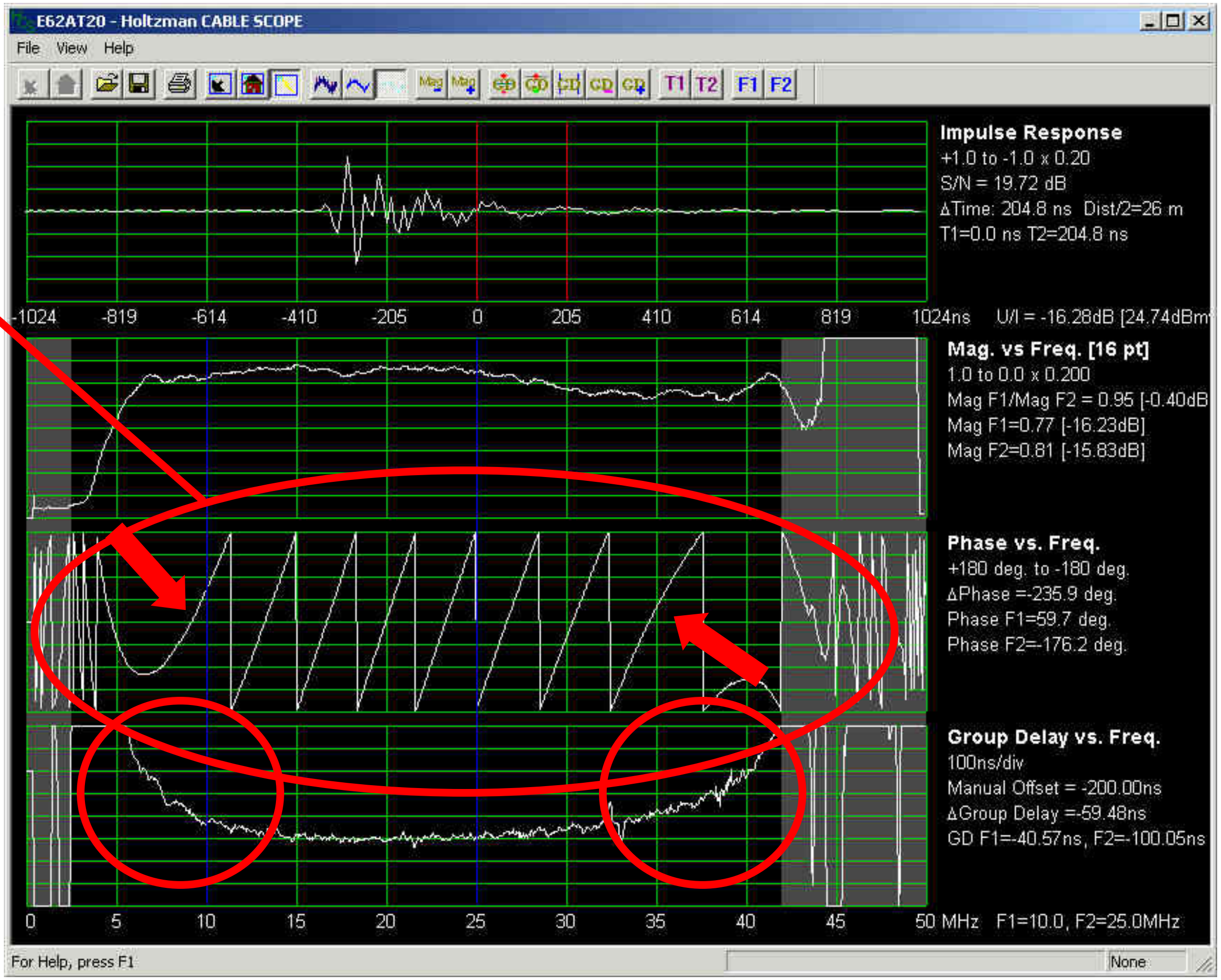


Phase Versus Frequency

- Another way of looking at this is to say that the cable's velocity of propagation is the same at all frequencies!
- In other words, every frequency takes 116.86 ns to travel from one end of the 100 ft piece of cable to the other end.
- But what happens if something in the signal path causes some frequencies to travel a little slower than other frequencies?

Phase Versus Frequency

- Take a look at the phase versus frequency plot on this screen shot
- Where phase is not linear versus frequency—that is, where the sloped line is not straight—group delay exists

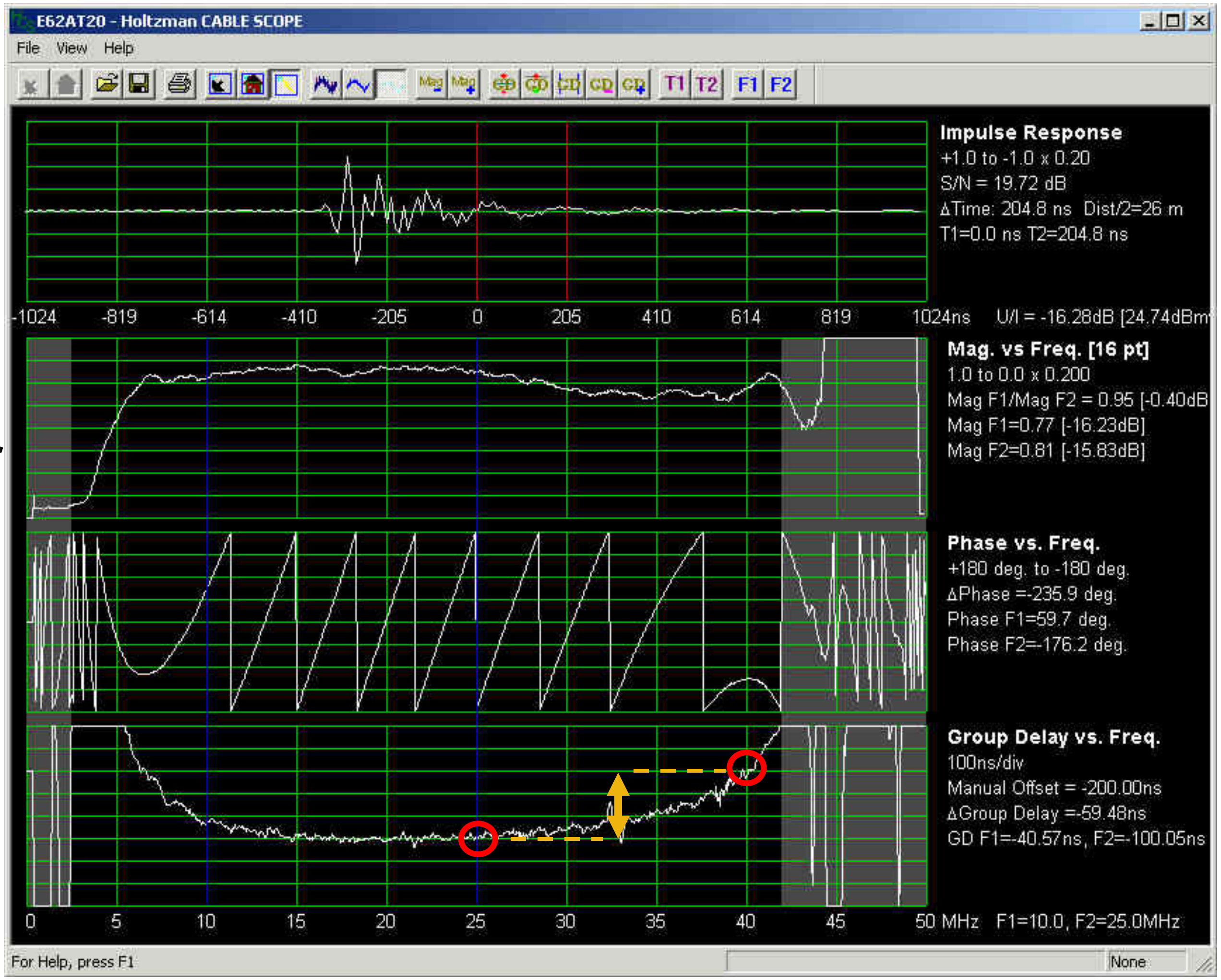


Graphic courtesy of Holtzman, Inc.

Phase Versus Frequency

- As before, the group delay between 25 and 40 MHz is about 300 ns (3 vertical divisions at 100 ns each)
- It takes the 40 MHz signal 300 ns longer to reach the headend than the 25 MHz signal

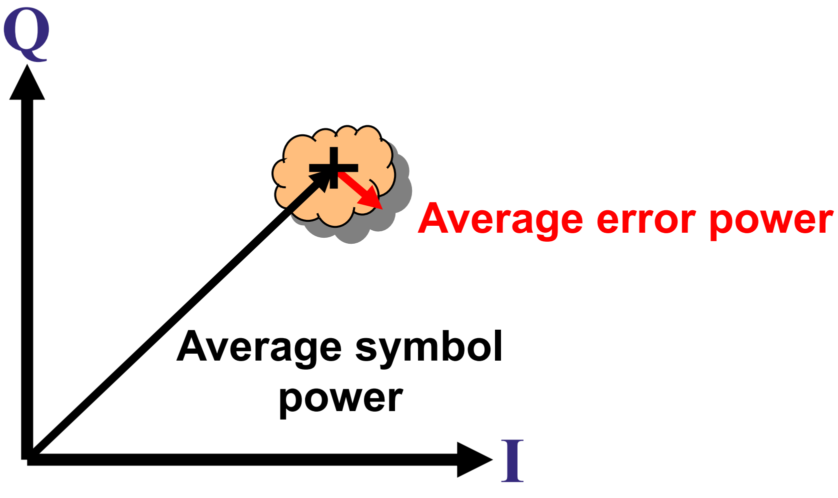
Time difference in nanoseconds



Graphic courtesy of Holtzman, Inc.

Why Is Group Delay Important?

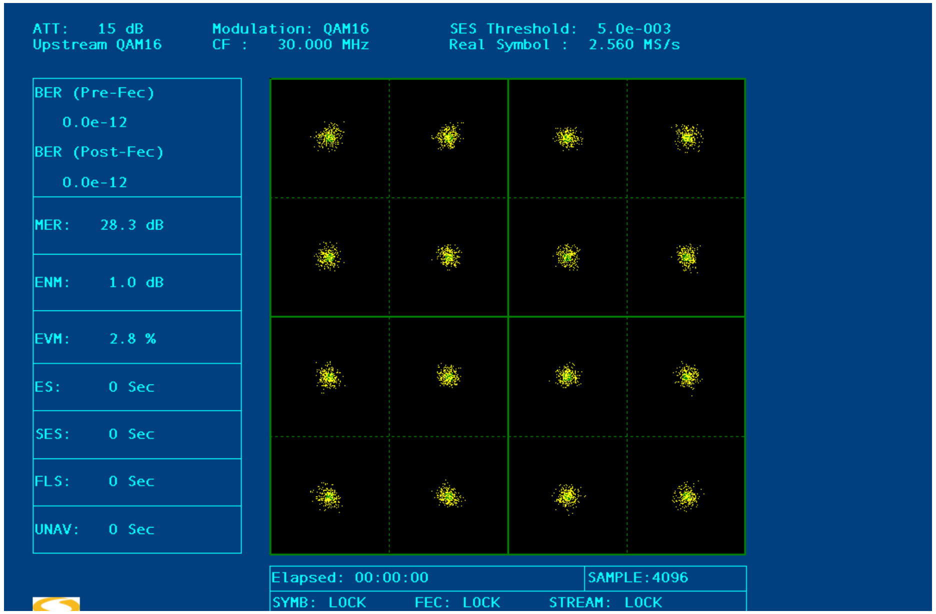
- Group delay, a linear distortion, causes inter-symbol interference to digitally modulated signals
- This in turn degrades receive modulation error ratio (RxMER)—the constellation symbol points get "fuzzy"



MER = 10log(average symbol power/average error power)

Why Is Group Delay Important?

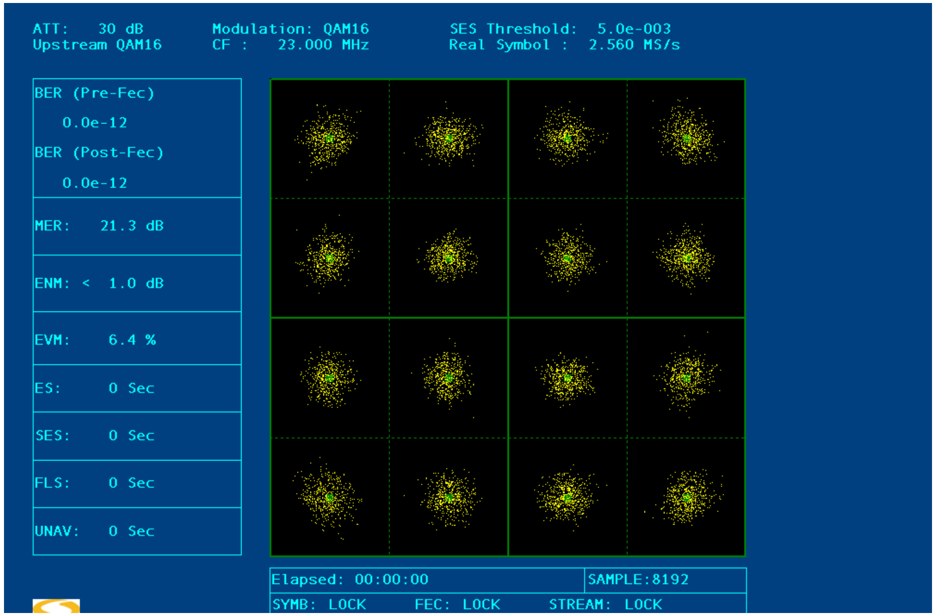
- **This test equipment screen shot is from a cable network's upstream spectrum in which in-channel group delay was negligible—about 60 to 75 ns peak-to-peak.**
- **Unequalized RxMER is 28.3 dB, well above the 17~20 dB RxMER failure threshold for 16-QAM.**



Graphic courtesy of Sunrise Telecom

Why Is Group Delay Important?

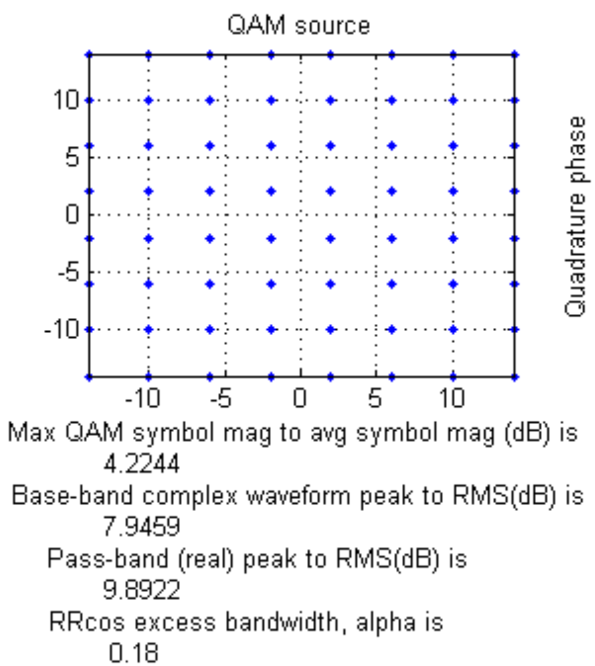
- **In this example, in-channel group delay was around 270 ns peak-to-peak.**
- **Unequalized RxMER is about 21 dB, very close to the 16-QAM failure threshold. 16-QAM would not work on this upstream!**

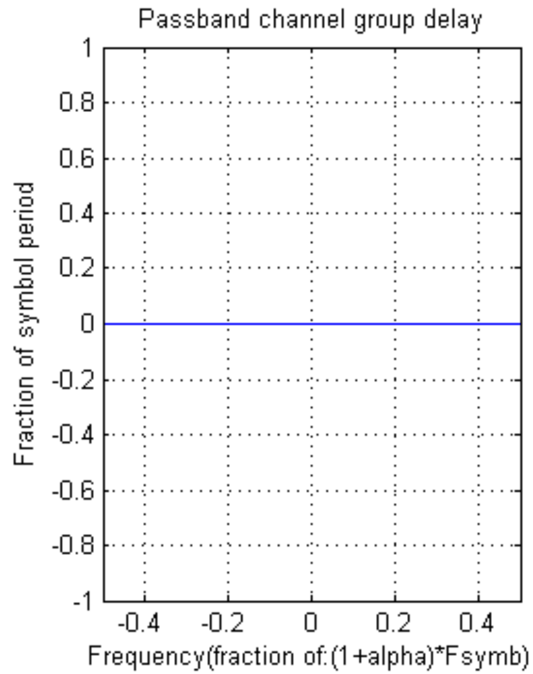


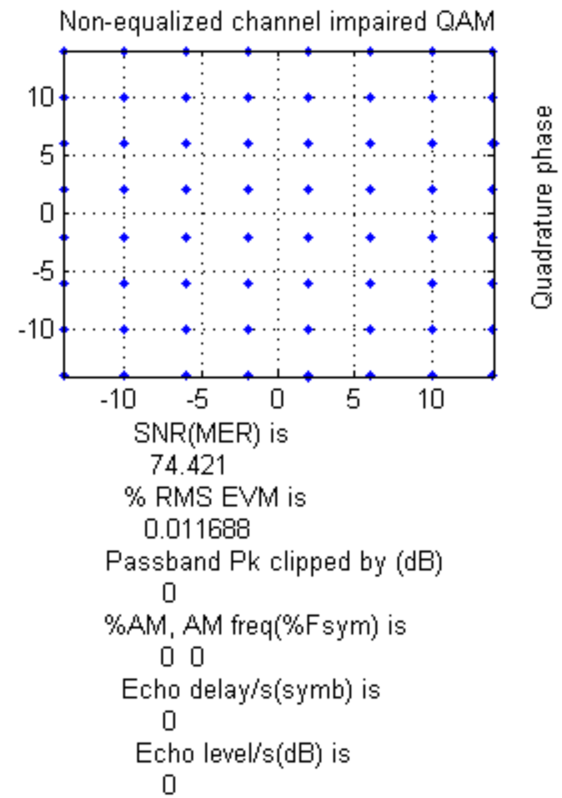
Graphic courtesy of Sunrise Telecom

Why Is Group Delay Important?

MATLAB® simulation for 64-QAM—no group delay

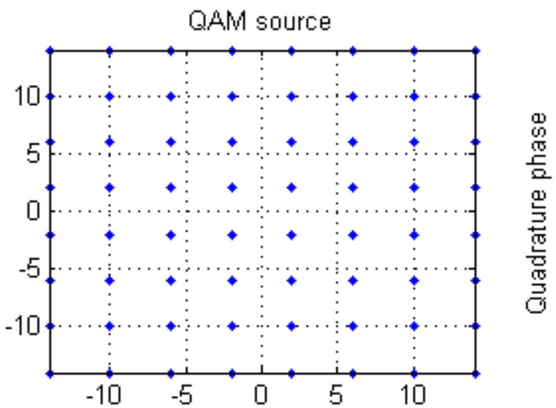




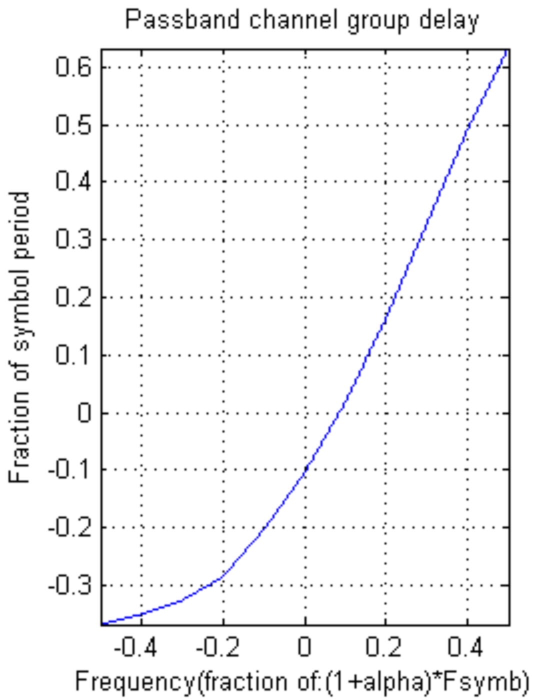


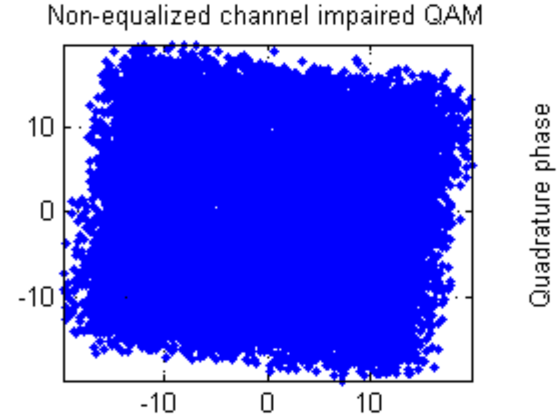
Why Is Group Delay Important?

MATLAB® simulation for 64-QAM—with group delay



Max QAM symbol mag to avg symbol mag (dB) is
4.2244
Base-band complex waveform peak to RMS(dB) is
7.9459
Pass-band (real) peak to RMS(dB) is
9.8922
RRcos excess bandwidth, alpha is
0.18





SNR(MER) is
12.228
% RMS EVM is
15.0447
Passband Pk clipped by (dB)
0
%AM, AM freq(%Fsym) is
0 0
Echo delay/s(symb) is
0
Echo level/s(dB) is
0

Wrapping Up

- **Common sources of group delay in a cable network**
  - AC power coils/chokes (mostly affects the 5 MHz to 10 MHz region in the upstream spectrum)
  - Node and amplifier diplex filters (affect frequencies near the diplex filter cutoff region in the upstream and downstream)
  - Band edges and rolloff areas
  - High-pass filters, data-only filters, step attenuators, taps or in-line equalizers with filters
  - Group delay ripple caused by impedance mismatch-related micro-reflections and amplitude ripple (“standing waves,” or poor frequency response)

Wrapping Up

- **The Fix?**

  Use adaptive pre-equalization available in DOCSIS 1.1 and later modems (generally not supported in DOCSIS 1.0 modems)

  Avoid frequencies where diplex filter group delay variation is common (if those frequencies in the upstream are used, be sure that upstream pre-equalization is turned on)

  Sweep the forward and reverse to ensure frequency response is flat (set equipment to highest resolution available; use resistive test points or probe seizure screws to see amplitude ripple)

  Identify and repair impedance mismatches that cause micro-reflections

  Use specialized test equipment to characterize and troubleshoot group delay variation (group delay variation can exist even when frequency response is flat)

References

- Farmer, J., D. Large, W. Ciciora and M. Adams. *Modern Cable Television Technology: Video, Voice and Data Communications, 2nd Ed*., Morgan Kaufmann Publishers; 2004
- Freeman, R. *Telecommunications Transmission Handbook, 4th Ed*., John Wiley & Sons; 1998
- Hranac, R. "Group delay" *Communications Technology*, January 1999

  Williams, T. "Tackling Upstream Data Impairments, Part 1" *Communications Technology*, November 2003

References

- **Williams, T. "Tackling Upstream Data Impairments, Part 2" *Communications Technology*, December 2003**

  Hranac, R. "Microreflections and 16-QAM" *Communications Technology*, March 2004

  Hranac, R. "Linear Distortions, Part 1" *Communications Technology*, July 2005

  Hranac, R. "Linear Distortions, Part 2" *Communications Technology*, August 2005

Q and A